AP® Calculus BC
2002 Sample Student Responses

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CALCULUS

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$\int_{.5}^{1} (e^x - \ln x)\,dx = 1.223$ units^2

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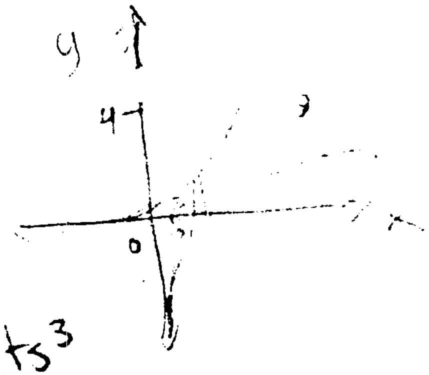
A2



Work for problem 1(b)

$R = 4 - \ln x$

$r = 4 - e^x$

$\pi\int_{.5}^{1}\left((4-\ln(x))^2 - (4-e^x)^2\right)dx = 23.609$ units^3

Work for problem 1(c)

$h(x) = f(x) - g(x)$

$= e^x - \ln x$

$h'(x) = e^x - \frac{1}{x}$

For criticals

$e^x - \frac{1}{x} = 0$

$\ln e^x = \frac{1}{x}$

$x = \ln(x)$

$x = -\ln x$

$\ln x + x = 0$

$x = .567$

End pts: .5, 1

x	f(x)
.5	2.341
.567	2.330
1	2.718

To determine the absolute minimum and maximum I found any criticals (when $h'(x)$ equals 0) and the end points. There was only one critical number, which occurred at x=.567. When I compared the values of each number (see chart), I found the absolute minimum value to be 2.330 and the absolute maximum value to be 2.718 (or e).

GO ON TO THE NEXT PAGE.

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CALCULUS

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$$A = \int_{\frac{1}{2}}^{1} e^x - \ln x \, dx = \boxed{1.223} \; u^2$$

(using fnInt)

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C2

Work for problem 1(b)

$$V = \pi\int_{\frac{1}{2}}^{1} (4-\ln x)^2-(4-e^x)^2\,dx = \boxed{23.610}\ u^3$$

(using fnInt)

Work for problem 1(c)

$$h'(x) = f'(x) - g'(x)$$

$$h'(x) = e^x - \frac{1}{x} = 0$$

$$h(1) = \boxed{e - \ln 1} = MAX$$

e^x grows faster than $\ln x$ so on the interval $\frac{1}{2} \leq x \leq 1$, the greatest value of $h(x)$ will be at $x=1$.

$$h\left(\tfrac{1}{2}\right) = \boxed{e^{\frac{1}{2}} - \ln \tfrac{1}{2}} = MIN$$

since e^x grows faster than $\ln x$ the min. value will be at the very beginning of the interval at $x=\frac{1}{2}$.

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Work for problem 2(a)

$\int_9^{17} E(t)\,dt = \int_9^{17} 15600/(t^2-24t+160)\,dt = 6004.27$

$\boxed{\approx 6004 \text{ people}}$

Work for problem 2(b)

$\int_9^{23} \left(\frac{15600}{t^2-24t+160}\right) dt$ = TOTAL ENTERED

= 7275.55 ≈ 7276 PEOPLE

$$\begin{array}{r} 7276 \\ -\ 6004 \\ \hline \end{array}$$

AFTER 5 → 1272

$\times$ \$11

\$13,992

BEFORE 5 6004

$\times$ \$15

\$90,060

\$13,992 + \$90,060 = \$104,052 made on the given day

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Work for problem 2(c)

$$H(17) = \int_9^{17} \left(\frac{15600}{t^2 - 24t + 160}\right) - \left(\frac{9890}{t^2 - 38t + 370}\right) dt = 3725$$

$$H'(17) = \left(\frac{15600}{(17^2) - 24(17) + 160}\right) - \frac{9890}{(17^2 - 38(17) + 370)}$$

$$= 380 - 760$$

$H'(17) = -380$ → This is the rate of change at 5 o'clock THAT PEOPLE ARE ENTERING THE PARK COMPARED TO THOSE LEAVING THE PARK MORE PEOPLE LEAVING THAN ENTERING AT $t = 17$

$H(17) = 3725$ → THIS is THE AMOUNT OF PEOPLE INSTANTANEOUSLY AT THE PARK.

Work for problem 2(d)

$$H'(t) = \frac{15600}{(t^2 - 24t + 160)} - \frac{9890}{t^2 - 38t + 370} = 0$$

$$t = 15.7948$$

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Work for problem 2(a)

$$\int_9^{17} \frac{15600}{(t^2-24t+160)}\,dt = 6004 \text{ people}$$

Work for problem 2(b)

$$15\int_9^{17} \frac{15600}{(t^2-24t+160)}\,dt + 11\int_{17}^{23} \frac{9890}{(t^2-38t+370)}\,dt$$

$$90064 + 54950$$

$$\$\,115{,}014$$

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Work for problem 2(c)

$H'(t) = E(t) - L(t)$

$H'(17) = 380.4878 - 760.7692$

$H'(17) = -452.2814$

H(17) represents the number of people in the park at t=17. H'(17) represents the rate at which the population of the park is changing at t=17

Work for problem 2(d)

$E(t) - L(t) = 0$

$\frac{15600}{(t^2 - 24t + 160)} - \frac{9890}{t^2 - 38t + 370} = 0; \; t = \boxed{15.79481}$

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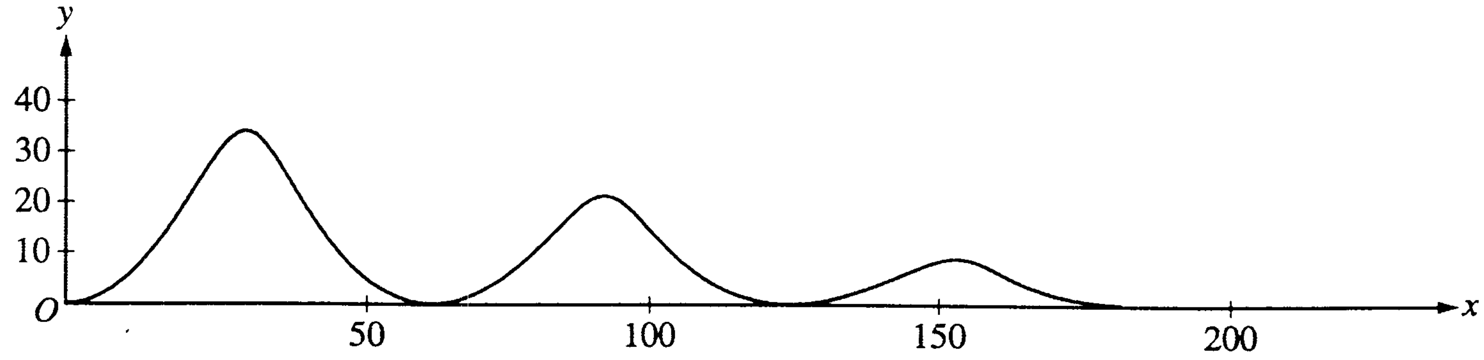
B1



Work for problem 3(a)

$x'(t) = 10 + 4\cos t$

$y'(t) = (20-t)\sin t + \cos t - 1$

$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} \qquad \frac{y'(2)}{x'(2)} = \text{slope}$

$$\frac{(20-2)\sin(2) + \cos(2) - 1}{10 + 4\cos(2)} = 1.794$$

Work for problem 3(b)

$x(t) = 10t + 4\sin t$

$10t + 4\sin t = 140$

$t = 13.647$ seconds

$x'(t) = 10 + 4\cos t$

$x''(t) = -4\sin t$

$y'(t) = (20-t)\sin t + \cos t - 1$

$y''(t) = -\sin t + (20-t)\cos t - \sin t$

$x''(13.647) = -3.529$

$y''(13.647) = 1.226$

$\overrightarrow{a(t)} = \langle -3.529, 1.226 \rangle$

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B_2

Work for problem 3(c)

$y(t) = (20-t)(1-\cos t)$

$y'(t) = (20-t)\sin t + \cos t - 1$

$y'(t) = 0$ $\quad$ $x'(t) = 10+4\cos t$

$t = 3.024$ s

$$\text{speed} = \sqrt{(x'(3.024))^2 + (y'(3.024))^2}$$

speed = 6.028 m/s

Work for problem 3(d)

$y(t) = (20-t)(1-\cos t)$ $\quad$ $x'(t) = 10+4\cos t$

$y'(t) = (20-t)(\sin t) + \cos t - 1$

$y(t) = 0$

$t = 6.283$ and 12.566

$$\text{Avg speed} = \frac{\int_{6.283}^{12.566} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt}{12.566 - 6.283}$$

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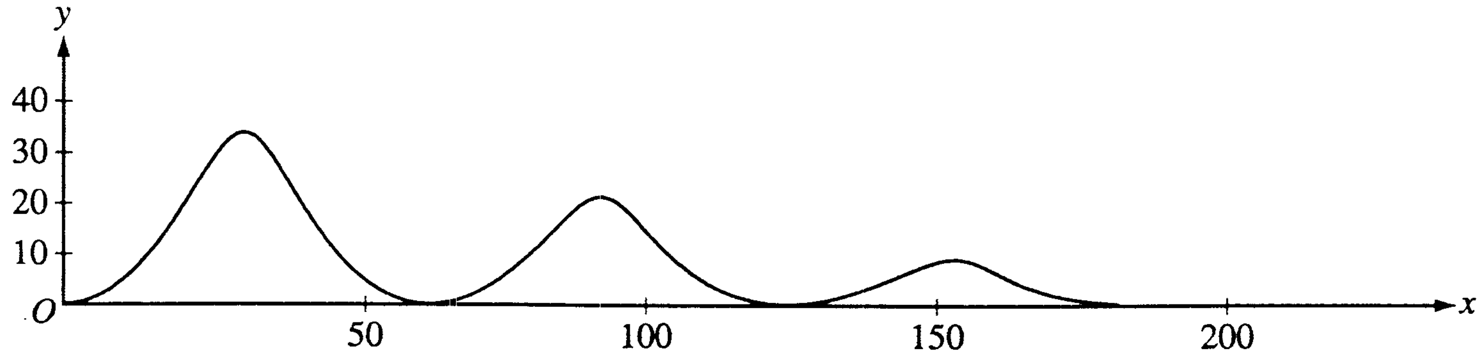
D1



Work for problem 3(a)

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{(20-t)\sin t + \cos t - 1}{10 + 4\cos t}\Bigg|_{t=2} = 1,793$$

Work for problem 3(b)

$a(t) = \langle x''(t), y''(t)\rangle$

$x''(t) = -4\sin t$

$y''(t) = -\sin t + (20-t)\cos t - \sin t$

$= (20-t)\cos t - 2\sin t$

$x(t) = 140 = 10t + 4\sin t$

$t = 13.6471$

$x''(13.6471) \approx -3.529$

$y''(13.6471) \approx 1,2257$

$\vec{a} = \langle -3.529, 1.2257\rangle$

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D_2

Work for problem 3(c)

$$y'(t) = (20-t)\sin t + \cos t - 1 = 0$$
$$t = 3.02392$$

$$x'(3.02392) \approx 6.02766 \text{ m/sec}$$

(don't worry about $\sqrt{x'(t)^2 + y(t)^2}$ because
$y'(t) = 0$)

Work for problem 3(d)

$$y(t) = (20-t)(1-\cos t) = 0$$
$$t = \cancel{20}$$
$$\cos t = 1$$
$$t = \cancel{0}, 2\pi, 4\pi, \cancel{6\pi}$$

$$\frac{\sqrt{[y(\cancel{4}\pi)]^2 + [x(4\pi)]^2} - \sqrt{[\cancel{y\ 2\pi}]^2 + [x(2\pi)]^2}}{4\pi - 2\pi} = \frac{x(4\pi) - x(2\pi)}{2\pi} \text{ m/sec}$$

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B1

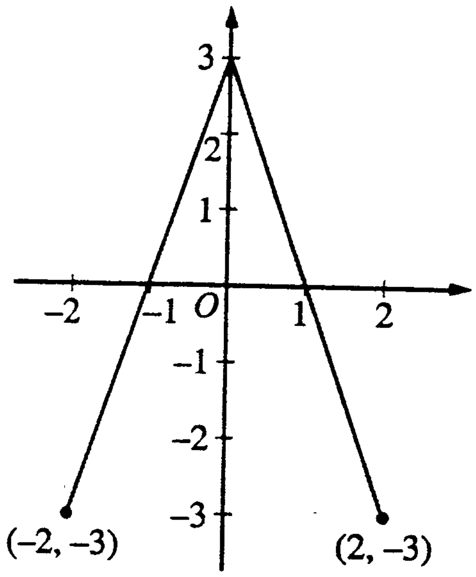
CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f

Work for problem 4(a)

$g(-1) = \int_0^{-1} f(t) = -1 \cdot 3 \cdot \frac{1}{2} = \frac{-3}{2}$

$g'(-1) = f(-1) = 0$

$g''(-1) = f'(-1) = 3.$

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B_a

Work for problem 4(b)

$g'(x) = f(x)$.

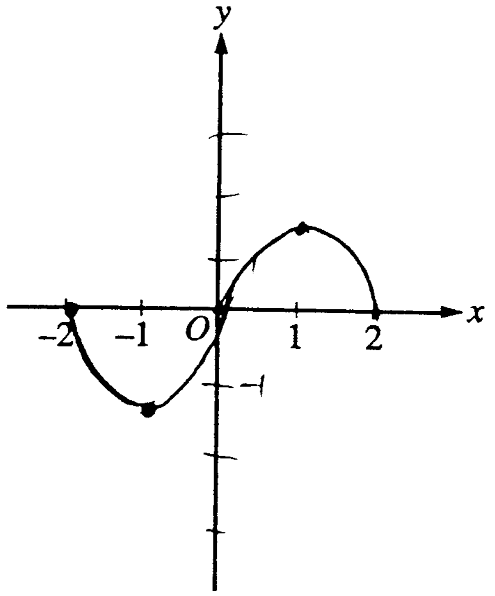
For $-1 < x < 1$, g is increasing, because $g'(x) = f(x)$ is positive for this interval.

Work for problem 4(c)

$g''(x) = f'(x)$

For $0 < x < 2$, g concaves down, because $g''(x) = f'(x)$ is negative for this interval.

Work for problem 4(d)



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C1

NO CALCULATOR ALLOWED

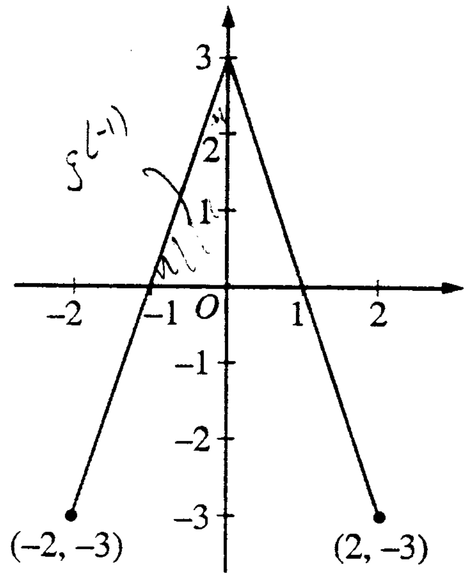
CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f

Work for problem 4(a)

$g(x) = \int_0^x f(t)\,dt$

$g(-1) = \int_0^{-1} f(t)\,dt = \frac{1}{2}bh = \frac{1}{2}(1)(3) = \frac{3}{2}$

$\boxed{g(-1) = \frac{3}{2}}$

$g'(-1) = f(t) = 0$

$\boxed{g'(-1) = 0}$

$g''(-1) =$ slope from -2 to 0 $= \frac{-3-3}{-2-0}$ $\frac{-6}{-2} = 3$

$\boxed{g''(-1) = 3}$

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NO CALCULATOR ALLOWED

C2

Work for problem 4(b)

$g'(x) = f(x)$

$g(x)$ increases when $f(x)$ is positive $f(x) > 0$ at $-1 < x < 1$

$g(x)$ increases at $-1 < x < 1$

Work for problem 4(c)

$g(x)$ is concave down where $g''(x)$ is negative

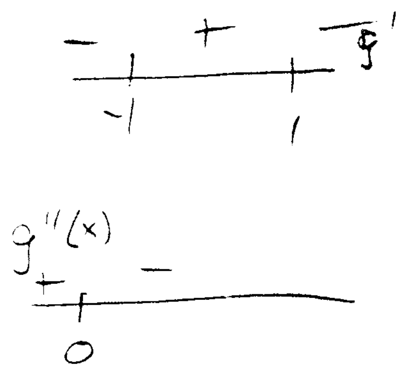
$g'(x) = f(x)$

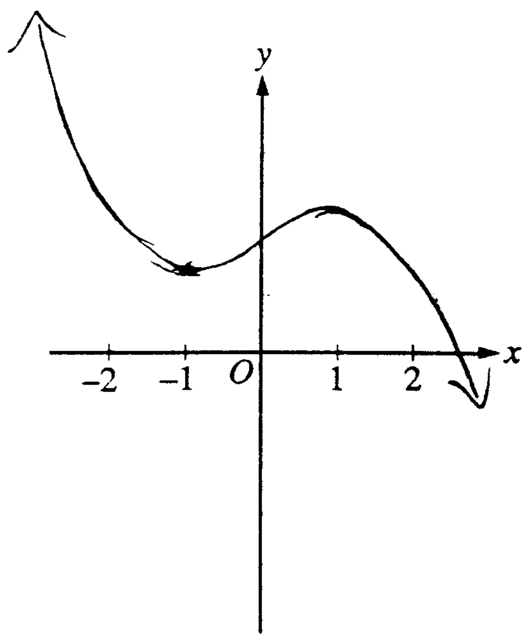
$g''(x) = f'(x)$ $g''(x) = f'(x)$ is negative for all $0 < x < 2$

$f'(x) = \frac{-3-3}{2-0} = -3$

$g(x)$ is concave down for all $0 < x < 2$

Work for problem 4(d)





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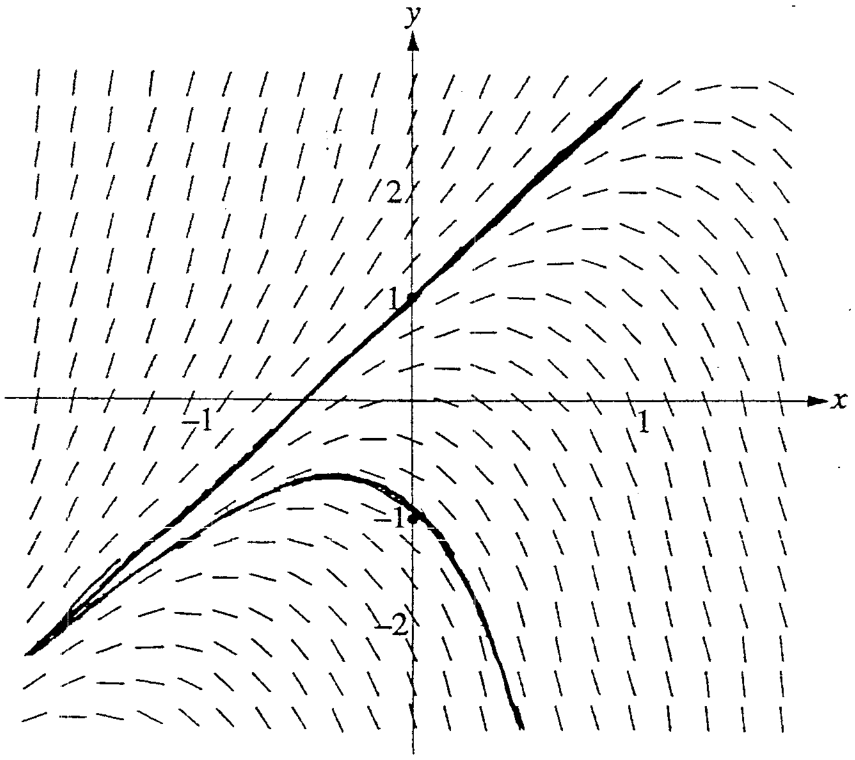
B1

Work for problem 5(a)



Work for problem 5(b)

$\frac{dy}{dx} = 2y - 4x$

x	f(x)	f'(x)
0	1	2
0.1	1.2	2
0.2	1.4	

$2(1) - 4(0)$ $\quad 1 + (2 \cdot 0.1) = 1.2$

$2(1.2) - 4(0.1)$ $\quad 1.2 + (2 \cdot 0.1) = 1.4$

$\boxed{f(0.2) \approx 1.4}$

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B2

Work for problem 5(c)

$\frac{dy}{dx} = 2y - 4x = 2$

~~$dy = (2y-4x)dx$~~

$y = \frac{2+4x}{2}$

$y = 2x + 1$

$\boxed{b = 1}$

Work for problem 5(d)

$g'(x) = \frac{dy}{dx} @(0,0) = 2(0) - 4(0) = 0$ critical point

$g''(x) = 2(2y-4x) - 4 = 4y - 8x - 4$

~~$g''(x) =$~~ $g''(x) @(0,0) = 4(0) - 8(0) - 4 = -4$ negative; g is concave down

$\boxed{\text{g has a local-maximum at } (0,0)}$

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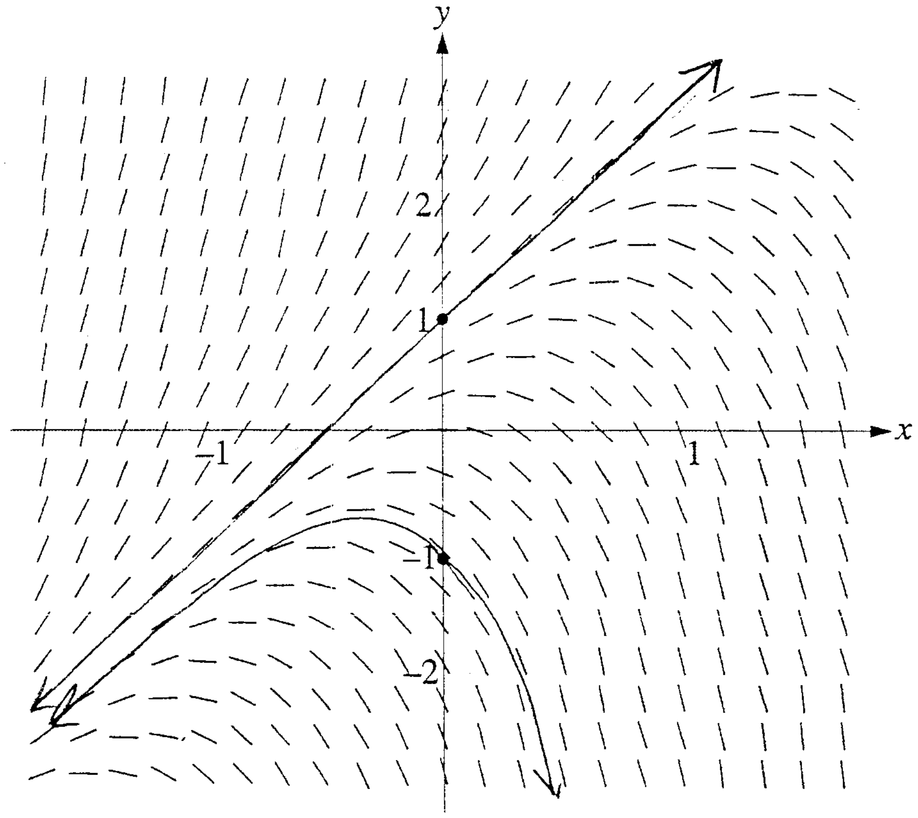
D1

Work for problem 5(a)

$\frac{dy}{dx} = 2y - 4x$



Work for problem 5(b)

$f(0) = 1$

x	y	dx	f'(x)	dy
0	1	0.1	2	0.2
0.1	1.2	0.1	2	0.2
0.2	$\boxed{1.4}$			

$f(0.2) \approx 1.4$ using Euler's method.

$\left.\frac{dy}{dx}\right]_{x=0,y=1} = 2(1) - 4(0) = 2$

$dy = f'(x)\,dx$

$\left.\frac{dy}{dx}\right]_{x=0.1,y=1.2} = 2(1.2) - 4(0.1)$

$= 2.4 - 0.4 = 2$

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1D2

Work for problem 5(c)

$y = 2x + b$

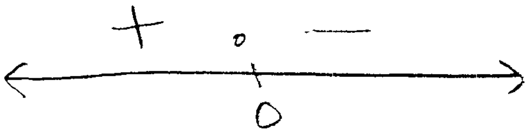
When the solution to $\frac{dy}{dx}$ passes through (0,1), the resulting line has the equation $y = 2x + 1$. Therefore, $b = 1$ when $y = 2x + b$ is a solution to this diff. equation.

Work for problem 5(d)

$g(0) = 0$

$g'(x) = \frac{dy}{dx} = 2y - 4x$

g'(x):



$\left.\frac{dy}{dx}\right]_{x=0,y=0} = 0$

$\left.\frac{dy}{dx}\right]_{x=-1,y=0} = -4(-1) = 4$

$\left.\frac{dy}{dx}\right]_{x=1,y=0} = -4(1) = -4$

There is a local maximum at (0,0) on the graph of g, since g'(x) is increasing then decreasing on either side of x=0 (see sign chart).

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AP® Calculus BC
2002 Sample Student Responses

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B1

Work for problem 6(a)

(a) $$\sum_{n=0}^{\infty} \frac{(2x)^{n+1}}{n+1}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\overset{2x}{\cancel{(2x)^{n+2}}}}{(n+2)}\cdot\frac{n+1}{\cancel{(2x)^{n+1}}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(2x)(n+1)}{n+2}\right| = |2x|\cdot\lim_{n\to\infty}\left|\frac{n+1}{n+2}\right| < 1$$

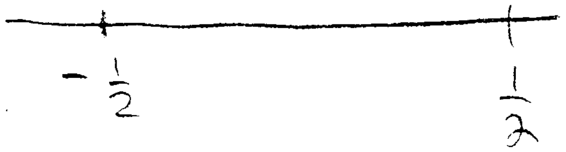
$$|2x|(1) < 1$$

$$|2x| < 1$$

$$-1 < 2x < 1$$

$$-\frac{1}{2} < x < \frac{1}{2}$$

$$\boxed{-\frac{1}{2} \le x < \frac{1}{2}}$$



$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n+1}$$

$$\lim_{n\to\infty}|a_n| = \lim_{n\to\infty}\left|\frac{1}{n+1}\right| = 0 \quad \text{(ii)} ✓$$

$$\left|\frac{a_{n+1}}{a_n}\right| < 1$$

$$\left|\frac{\frac{1}{n+2}}{\frac{1}{n+1}}\right| = \left|\frac{n+1}{n+2}\right| < 1 ✓$$

Converges by the alternating series test

$$\sum_{n=0}^{\infty} \frac{(1)^{n+1}}{n+1} = \sum_{n=0}^{\infty} \frac{1}{n+1}$$

$$\int_0^{\infty} \frac{1}{n+1}\,dn$$

$$= \lim_{b\to\infty}\int_0^b \frac{1}{n+1}\,dn$$

$$= \lim_{b\to\infty} \ln|n+1|\Big]_0^b$$

$$= \lim_{b\to\infty} \ln|b+1| - \ln|1|$$

$$= \infty - 0$$

divergent by the integral test

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B_2

Work for problem 6(b)

$n=0 \quad n=1 \quad n=2 \quad n=3$

$$f'(x) \approx 2 + \frac{8x}{2} + \frac{24x^2}{3} + \frac{64x^3}{4} + \dots$$

$$\boxed{f'(x) \approx \quad 2 + 4x + 8x^2 + 16x^3 + \dots + 2^{(n+1)}x^n}$$

$n=0 \quad n=1 \quad n=2 \quad n=3$

$2^{n+1} \quad 2^2 \quad 2^3 \quad 2^4$

Work for problem 6(c)

$$f'\left(-\frac{1}{3}\right) \approx 2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27} + \dots + 2^{n+1}\left(-\frac{1}{3}\right)^n$$

$$\sum_{n=0}^{\infty} 2^{n+1}\left(-\frac{1}{3}\right)^n = \sum_{n=0}^{\infty} \frac{2^n \cdot 2 \cdot (-1)^n}{3^n} = \sum_{n=0}^{\infty} 2 \cdot \left(\frac{-2}{3}\right)^n$$

$a = 2$

$r = -\frac{2}{3}$

$$S = \frac{a}{1-r} = \frac{2}{1-\left(-\frac{2}{3}\right)} = \frac{2}{\frac{5}{3}} \cdot \frac{3}{3} = \boxed{\frac{6}{5}}$$

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C1

Work for problem 6(a)

$$\frac{(2x)^{n+1}}{n+1}$$

$$\lim_{n\to\infty} \frac{(2x)^{(n+1)+1}}{(n+1)+1} \cdot \frac{n+1}{(2x)^{n+1}}$$

$$\lim_{n\to\infty} \frac{(2x)^{\overset{1}{\cancel{n+2}}}}{n+2} \cdot \frac{n+1}{(2x)^{\cancel{n+1}}}$$

$$\lim_{n\to\infty} \frac{(2x)\cancel{(n+1)}}{\cancel{(n+2)}}$$

$$-1 < 2x < 1$$

$$-\tfrac{1}{2} < x < \tfrac{1}{2}$$

$$\sum_{n=0}^{\infty} \frac{2(\frac{1}{2})^{n+1}}{n+1} \qquad \sum_{n=0}^{\infty} \frac{2(-\frac{1}{2})^{n+1}}{n+1}$$

$$\sum_{n=0}^{\infty} \frac{(1)^{n+1}}{n+1} \qquad \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n+1}$$

converges when $x = \frac{1}{2}$

① ~~numbers~~ terms decrease

② ~~number~~ term → 0

③ signs alternate

converges when $x = -\frac{1}{2}$

$$-\frac{1}{2} \le x \le \frac{1}{2}$$

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C2

Work for problem 6(b)

$f(x) \approx 2x + \frac{4x^2}{2} + \frac{8x^3}{3} + \frac{16x^4}{4}$

$f'(x) \approx 2 + 4x + 8x^2 + 16x^3$

$$\boxed{f'(x) = 2 + 4x + 8x^2 + 16x^3 + \ldots + 2^{n+1}x^n + \ldots}$$

Work for problem 6(c)

~~$f'(x) = 2 + \frac{4(2)x}{2} + \frac{8(3)x^2}{3} + \frac{16(4)(x^3)}{4}$~~

~~$f'(-\frac{1}{3}) = 2 + 4(-\frac{1}{3}) + 8(+\frac{1}{9}) + 16(-\frac{1}{27})$~~

$$\sum_{n=0}^{\infty} 2^{n+1}x^n$$

$$\sum_{n=0}^{\infty} \underline{2^{n+1}}(-\tfrac{1}{3})^n$$

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